RF FUNDAMENTALS
BEAM LOADING

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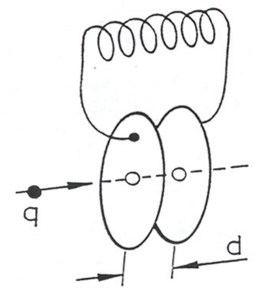
Equivalent Circuit for an rf Cavity

Simple LC circuit representing an accelerating resonator

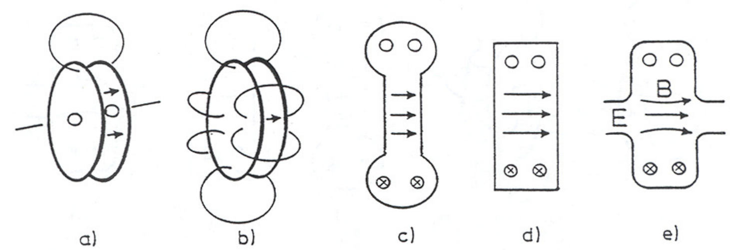
Metamorphosis of the LC circu into an accelerating cavity

Chain of weakly coupled pillbox cavities representing an accelerat cavity

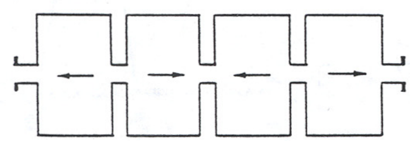
Chain of coupled pendula as its mechanical analogue



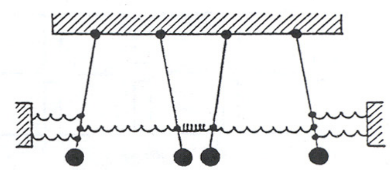
Simple lumped L-C circuit repesenting an accelerating resonator.
$\omega_o^2 = 1/LC$



Metamorphosis of the L-C circuit of Fig. 1 into an accelerating cavity (after R.P.Feynman[33]). Fig. 5d shows the cylindrical "pillbox cavity" and Fig. 5e a slightly modified pillbox cavity with beam holes (typical β between 0.5 and 1.0). Fig. 5c resembles a low β version of the pillbox variety ($0.2<\beta<0.5$).



Chain of weakly-coupled pillbox cavities representing an accelerating module



Chain of coupled pendula as a mechanical analogue to Fig. 6a

Parallel Circuit Model of an Electromagnetic Mode

- **Power dissipated in resistor *R*:** $P_{diss} = \frac{1}{2}\frac{V_c^2}{R}$

- **Shunt impedance:** $R_{sh} \equiv \frac{V_c^2}{P_{diss}} \qquad \Rightarrow \quad R_{sh} = 2R$

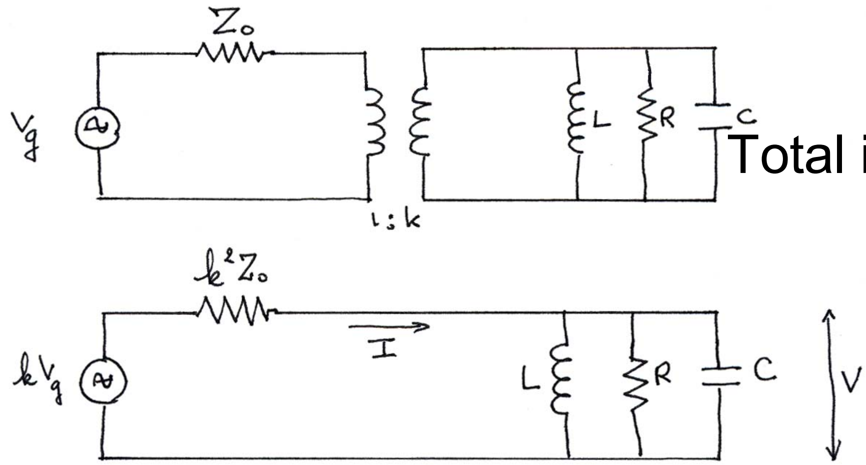
- **Quality factor of resonator:**

$$Q_0 \equiv \frac{\omega_0 U}{P_{diss}} = \omega_0 CR = \frac{R}{L\omega_c} = R\left(\frac{C}{L}\right)^{1/2}$$

$$\tilde{Z} = R\left[1 + iQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]^{-1}$$

$$\omega \approx \omega_0 \ , \qquad \tilde{Z} \approx R\left[1 + 2iQ_0\left(\frac{\omega - \omega_0}{\omega_0}\right)\right]^{-1}$$

1-Port System



Total impedance: $k^2 Z_0 + \dfrac{R}{1+2i\dfrac{Q_0}{\omega_c}\Delta\omega}$

$$I = \frac{kV_g}{k^2 Z_0 + \dfrac{R}{1+2i\dfrac{Q_0}{\omega_c}\Delta\omega}}$$

$$V = kV_g \frac{R}{R + k^2 Z_0 \;\; 1+2i\dfrac{Q_0}{\omega_c}\Delta\omega}$$

1-Port System

Energy content $U = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q_0}{\omega R} V^2$

$$= \frac{1}{2} \frac{Q_0}{\omega R} \; k^2 V_g^2 \; \frac{R^2}{\left(R + k^2 Z_0\right)^2 + 4k^4 Z_0^2 Q_0^2 \left(\frac{\Delta\omega}{\omega_C}\right)^2}$$

Incident power: $P_{inc} = \frac{V_g^2}{8Z_0}$

Define coupling coefficient: $\beta = \frac{R}{k_0^2 Z_0}$

$$\frac{U}{P_{inc}} = \frac{Q_0}{\omega_C} \frac{4\beta}{(1+\beta)^2} \frac{1}{1 + \left(\frac{2Q_0}{1+\beta}\right)^2 \left(\frac{\Delta\omega}{\omega_C}\right)^2}$$

1-Port System

Power dissipated

$$P_{diss} = \frac{\omega U}{Q_0} = P_{inc} \frac{4\beta}{(1+\beta)^2} \frac{1}{1+\left(\frac{2Q_0}{1+\beta}\right)^2 \left(\frac{\Delta\omega}{\omega_C}\right)^2}$$

Optimal coupling: $\frac{U}{P_{inc}}$ maximum or $P_{diss} = P_{inc}$

$\Rightarrow \Delta\omega = 0, \quad \beta = 1$: critical coupling

Reflected power

$$P_{ref} = P_{inc} - P_{diss} = P_{mc} \left[1 - \frac{4\beta}{(1+\beta)^2} \frac{1}{1+\left(\frac{2Q_0}{1+\beta} \frac{\Delta\omega}{\omega_C}\right)^2} \right]$$

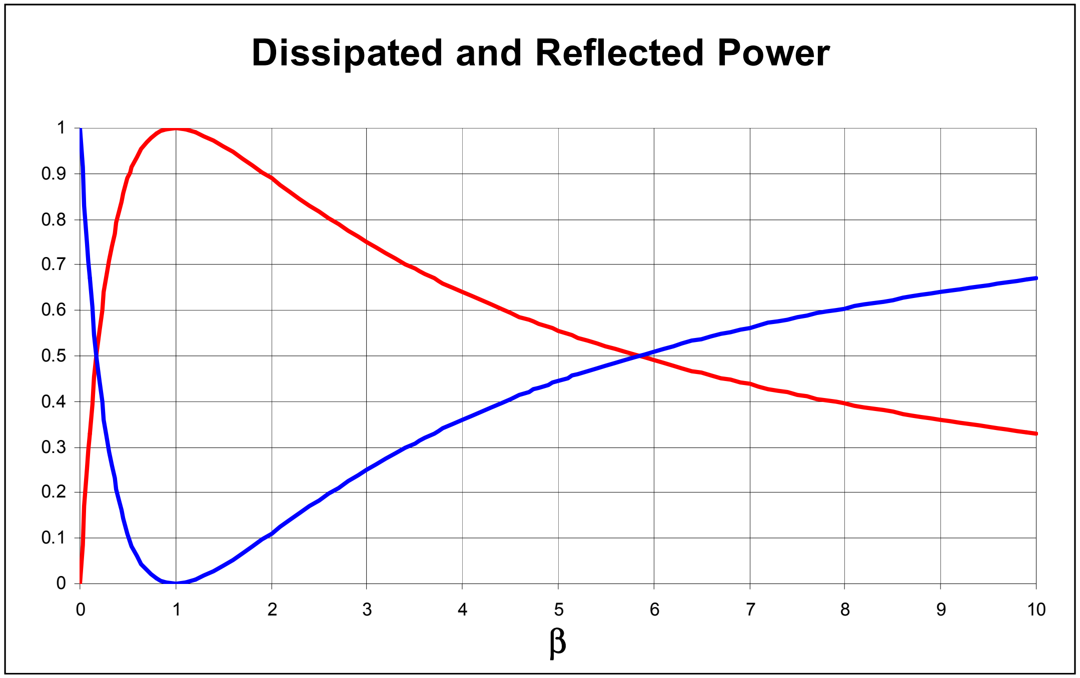
1-Port System

At resonance

$$U = \frac{Q_0}{\omega_C} \frac{4\beta}{(1+\beta)^2} P_{inc}$$

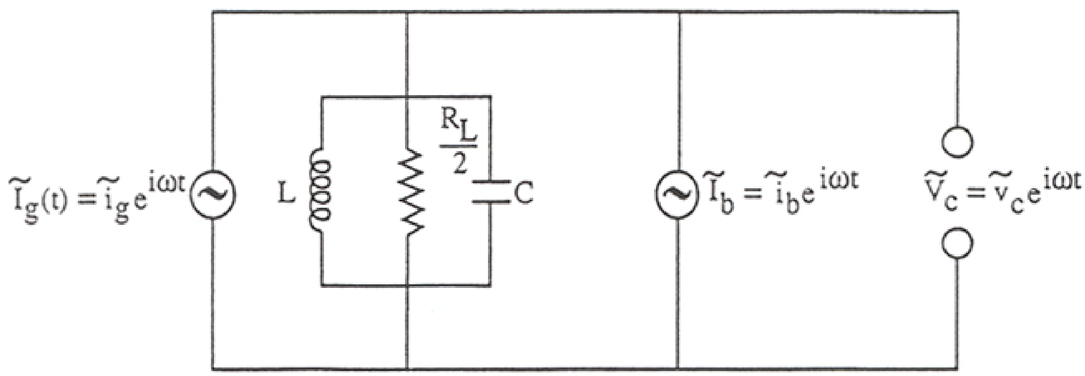
$$P_{diss} = \frac{4\beta}{(1+\beta)^2} P_{inc}$$

$$P_{ref} = \left(\frac{1-\beta}{1+\beta}\right)^2 P_{inc}$$



Equivalent Circuit for a Cavity with Beam

- **Beam in the rf cavity is represented by a current generator.**
- **Equivalent circuit:**



$$R_L = \frac{R_{sh}}{(1+\beta)}$$

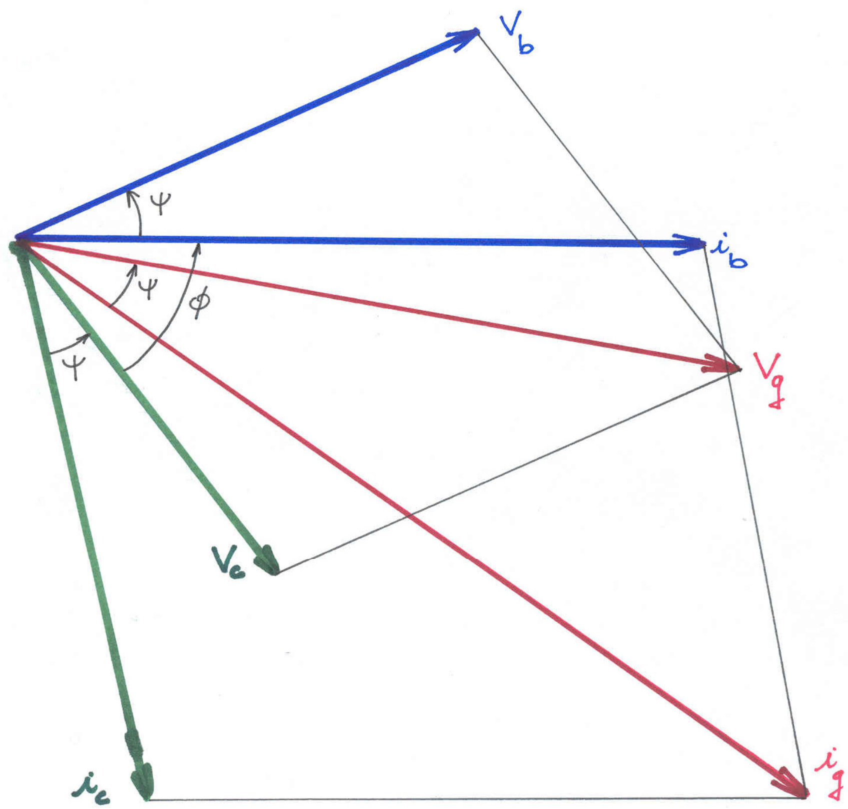
$\tilde{i}_b$ produces $\tilde{V}_b$ with phase ψ (detuning angle)

$\tilde{i}_g$ produces $\tilde{V}_g$ with phase ψ

$$\tilde{V}_c = \tilde{V}_g - \tilde{V}_b$$

$$\tan \psi = -2\frac{Q_0}{1+\beta}\frac{\Delta\omega}{\omega_0}$$

Equivalent Circuit for a Cavity with Beam



$$V_g = (P_g R_{sh})^{1/2} \frac{2\beta^{1/2}}{1+\beta} \cos\psi$$

$$V_b = \frac{i_b R_{sh}}{2(1+\beta)} \cos\psi$$

$$i_b = 2 i_0 \frac{\sin\frac{\theta_b}{2}}{\frac{\theta_b}{2}}$$

i_b: beam rf current

i_0: beam dc current

θ_b: beam bunch length

Equivalent Circuit for a Cavity with Beam

$$P_g = \frac{V_c^2}{R_{sh}} \frac{1}{4\beta} \left\{ (1+\beta+b)^2 + \left[(1+\beta)\tan\psi - b\tan\phi \right]^2 \right\}$$

$$b = \frac{\text{Power absorbed by the beam}}{\text{Power dissipated in the cavity}} = \frac{V_c i_0 \cos\phi}{\frac{V_c^2}{R_{sh}}} = \frac{R_{sh} i_0 \cos\phi}{V_c}$$

Minimize P_g:

$$(1+\beta_{opt})\tan\psi_{opt} = b\ \tan\phi$$

$$\beta_{opt} = |1+b|$$

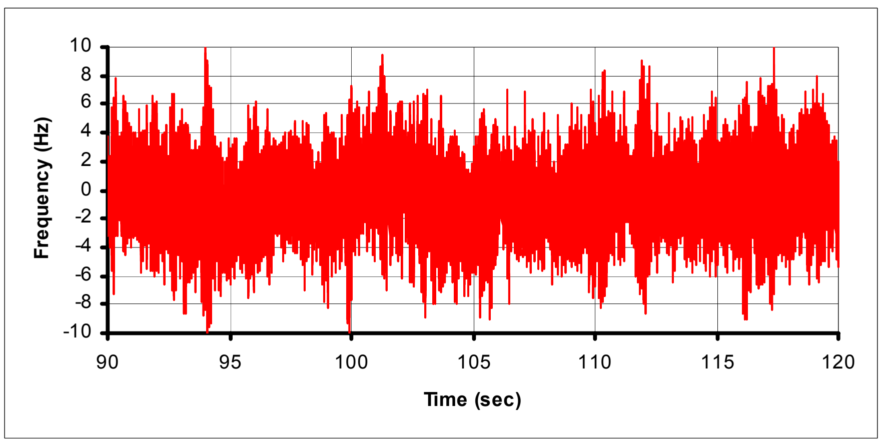
$$P_g^{opt} = \frac{V_c^2}{R_{sh}} \frac{|1+b| + (1+b)}{2}$$

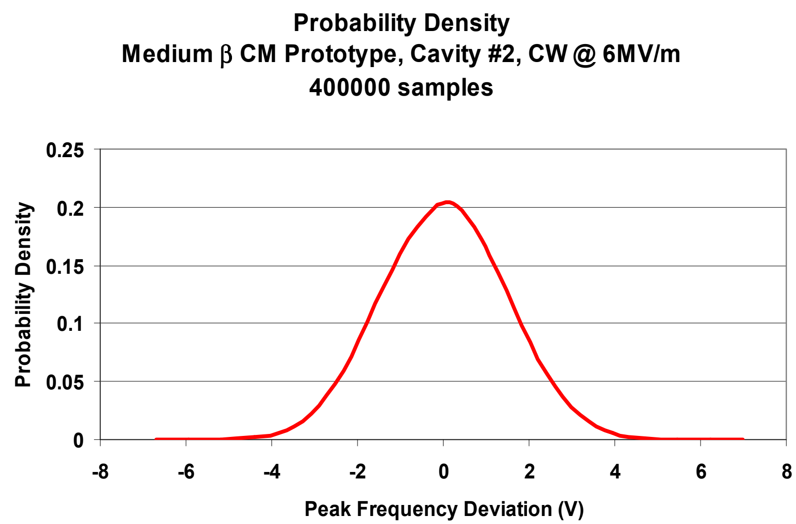
Cavity with Beam and Microphonics

- **The detuning is now** $\tan\psi = -2Q_L \frac{\delta\omega_0 \pm \delta\omega_m}{\omega_0}$ $\tan\psi_0 = -2Q_L \frac{\delta\omega_0}{\omega_0}$

  where $\delta\omega_0$ is the static detuning (controllable)

  and $\delta\omega_m$ is the random dynamic detuning (uncontrollable)





Q_{ext} Optimization with Microphonics

- Condition for optimum coupling:

$$\beta_{opt} = \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2}$$

and

$$P_g^{opt} = \frac{V_c^2}{2R_{sh}} \left[(b+1) + \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2} \right]$$

- In the absence of beam ($b=0$):

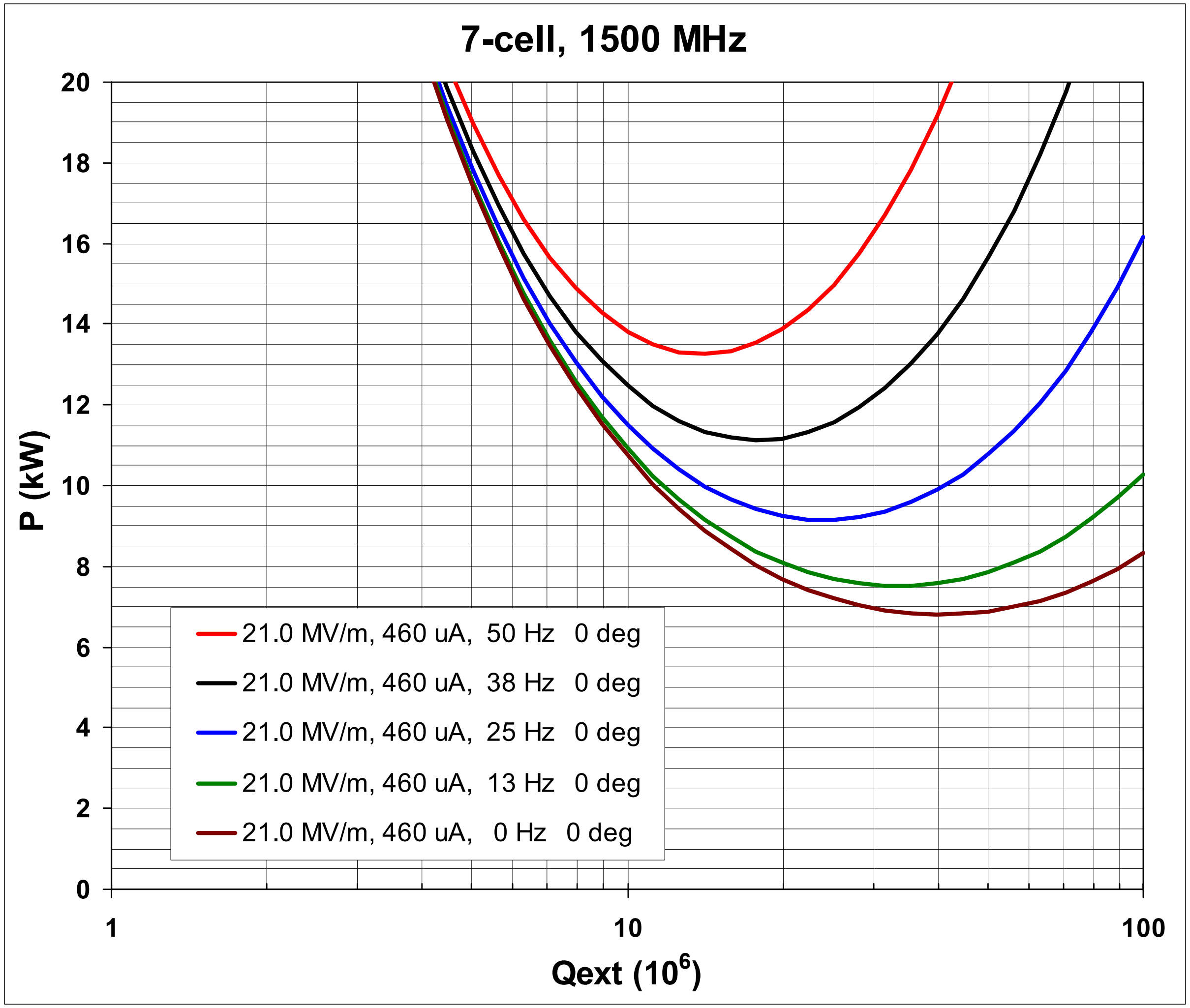
$$\beta_{opt} = \sqrt{1 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2}$$

and

$$P_g^{opt} = \frac{V_c^2}{2R_{sh}} \left[1 + \sqrt{1 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2} \right]$$

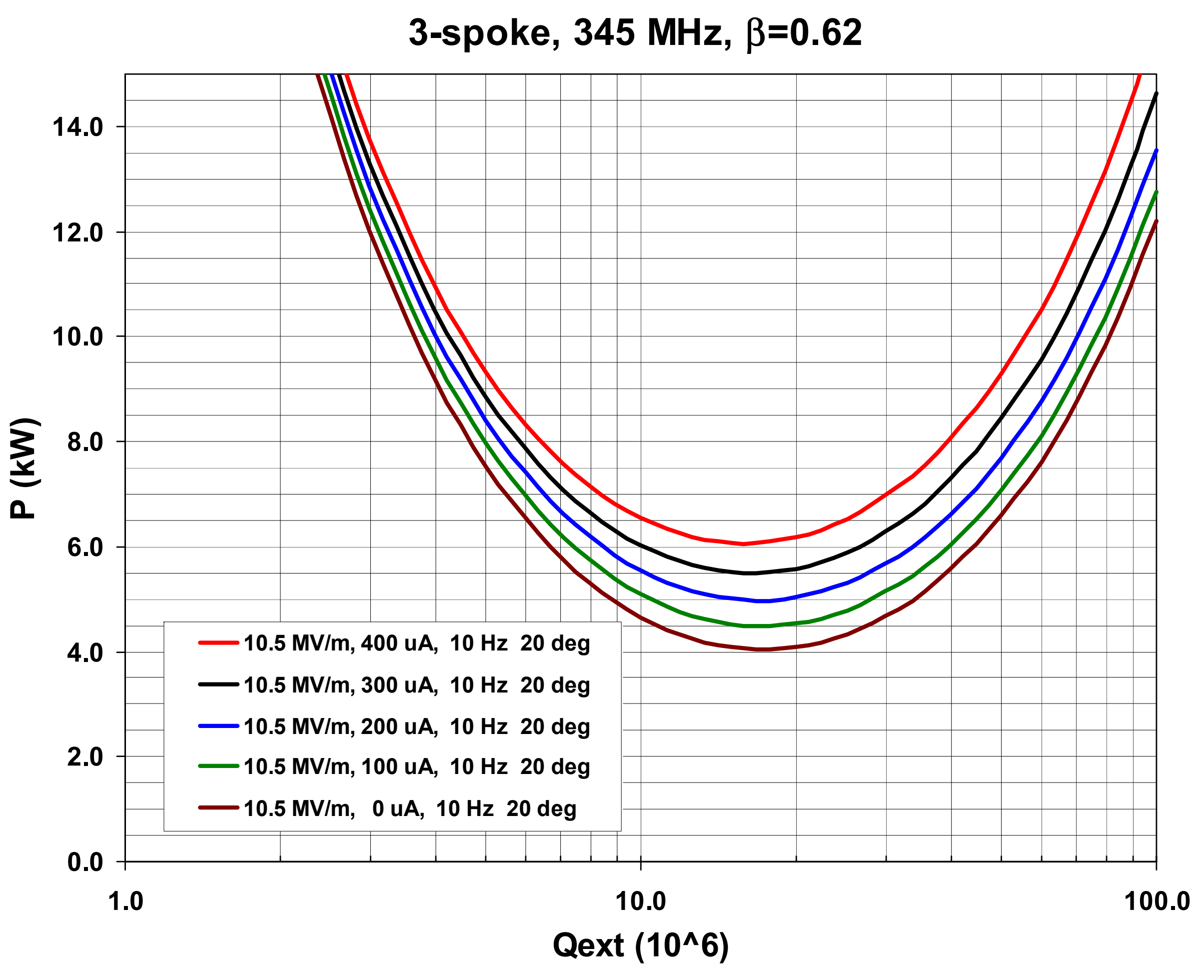
$\simeq U\ \delta\omega_m$ If $\delta\omega_m$ is very large

Example



7-cell, 1500 MHz
P (kW)
20
18
16
14
12
10
8
6
4
2
0
1
10
100
Qext (10^6)
21.0 MV/m, 460 uA, 50 Hz 0 deg
21.0 MV/m, 460 uA, 38 Hz 0 deg
21.0 MV/m, 460 uA, 25 Hz 0 deg
21.0 MV/m, 460 uA, 13 Hz 0 deg
21.0 MV/m, 460 uA, 0 Hz 0 deg

Example



3-spoke, 345 MHz, β=0.62
P (kW)
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
1.0
10.0
100.0
Qext (10^6)
10.5 MV/m, 400 uA, 10 Hz 20 deg
10.5 MV/m, 300 uA, 10 Hz 20 deg
10.5 MV/m, 200 uA, 10 Hz 20 deg
10.5 MV/m, 100 uA, 10 Hz 20 deg
10.5 MV/m, 0 uA, 10 Hz 20 deg

Example

- **ERL Injector and Linac:**

  δf_m=25 Hz, Q_0=1x10^{10} , f_0=1300 MHz, I_0=100 mA, V_c=20 MV/m, L=1.04 m, R_a/Q_0=1036 ohms per cavity

- **ERL linac: Resultant beam current, I_{tot} = 0 mA (energy recovery)**

  and β_{opt}=385 $\Rightarrow$ Q_L=2.6x10^7 $\Rightarrow$ P_g = 4 kW per cavity.

- **ERL Injector: I_0=100 mA and β_{opt}= 5x10^4 ! $\Rightarrow$ Q_L= 2x10^5 $\Rightarrow$ P_g = 2.08 MW per cavity!**

Note: I_0V_a = 2.08 MW $\Rightarrow$ optimization is entirely dominated by beam loading.

RF System Modeling

- To include amplitude and phase feedback, nonlinear effects from the klystron and be able to analyze transient response of the system, response to large parameter variations or beam current fluctuations

  – We developed a model of the cavity and low level controls using SIMULINK, a MATLAB-based program for simulating dynamic systems.

- Model describes the beam-cavity interaction, includes a realistic representation of low level controls, klystron characteristics, microphonic noise, Lorentz force detuning and coupling and excitation of mechanical resonances

RF System Model